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TOWARDS AN ASSESSMENT OF FAULT-TOLERANT DESIGN PRINCIPLES FOR SOFTWARE

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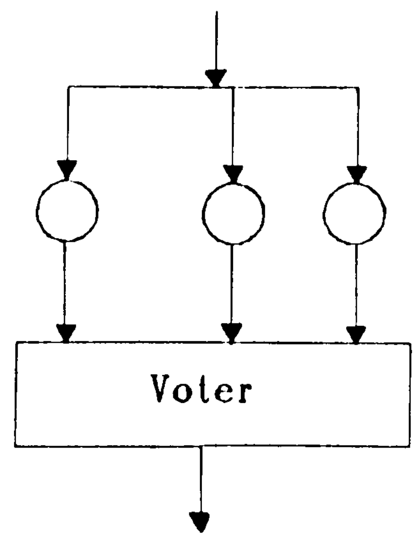
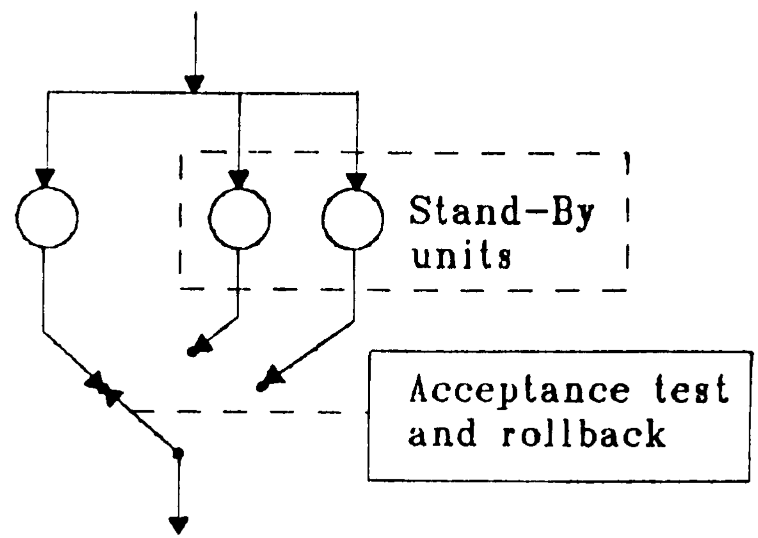
FAULT–TOLERANT SOFTWARE

Dissimilar, redundant software structured to reduce the probability of system failure due to software faults. Techniques address error detection and isolation, system recovery, and continued service.





HARDWARE:	N–Modular Redundancy	Stand–By Sparing
SOFTWARE:	N–Version Programming	Recovery Block

FAULT–TOLERANT SYSTEMS BRANCH: SOFTWARE RESEARCH SUPPORTED BY THE COMPUTER SCIENCE PROGRAM

DESIGN:

OPERATING SYSTEMS
- Investigation of fault–tolerant software for SIFT operating system (Brunelle, NASA LaRC)

SYSTEM THEORY
- Integration of control system–theoretic FDI Techniques (Caglayan, Charles River Analytics)

NEW TECHNOLOGY
- Automatic generation of FDI software (Wild, ODU)

RELIABILITY ASSESSMENT:

EXPERIMENTAL
- Investigation of independence assumption (Knight, UVA & Leveson, UCI)
- Investigation of test procedures for multi–version software (McAllister, NC State)

MODELS
- Assessment of generic software reliability models (Migneault, NASA LaRC)
- Development of basis for analysing strategy of software redundancy (Eckhardt, Lee, NASA LaRC)

THE BASIS FOR FAULT–TOLERANT SOFTWARE

- Software faults are assumed to be "independent" so that errors will be randomly distributed among replicate codes.
- Currently, there are research efforts to analyse this fundamental assumption.

HOWEVER

- Independence is not strictly needed for fault–tolerant software to be effective at reducing failure probability.
- It is a mathematically convenient assumption used to project the reliability of software fault–tolerant structures.

ASSESSING FAULT–TOLERANT SOFTWARE

If software errors are not randomly distributed, what is the impact on reliability?

Current state of the art does not provide answers.

Consider:

(1) Is an N–Version system of highly reliable components always more effective at reducing failure probability than a single version of software (on average)? If not, what causes this?

(2) What are the effects of different intensities of coincident errors on software redundancy?

(3) What is the effect of increasing N? Is there a limit to the effectiveness of software redundancy? Might an optimum value of N exist?

(4) Does the independence model provide a valid estimate of the failure probability of an N–Version system?

(5) Under what condition does independence hold?

GOAL OF CURRENT RESEARCH

Assess strategy of software redundancy

- population concepts
- sampling, sample size
- inference

AS OPPOSED TO:

Assessing an instance of fault-tolerance

- decide number of versions
- develop
- measure

QUANTITIES DESCRIBING COINCIDENT ERRORS MODEL

INPUT SPACE

COMPONENT POPULATION

	$\{ x_1$	x_2	x_3	. . .	$x_k \}$	
C_1	0	1	0	. . .	0	$\xrightarrow{v_1(x)}$
C_2	1	0	0	. . .	1	
⋮						
C_l	0	1	0	. . .	0	
⋮	$\downarrow$					
	$\theta(x_1)$					

CONDITIONAL FAILURE PROBABILITY

$$Q(F) = \int v_1(x)\, dQ$$

$$= \Pr\{ C_l \text{ fails} \}$$

Q = USAGE DISTRIBUTION

AVERAGE FAILURE PROBABILITY

$$E\left[\int V(x)\, dQ\right] = \int \theta(x)\, dQ$$

INTENSITY FUNCTION

$$\theta(x) = \Pr\{ V(x) = 1 \}$$

$$E[\, V(x)\,] = \theta(x)$$

INTENSITY DISTRIBUTION

$$G(y) = \int_{\{ x \,:\, \theta(x) \leq y \}} dQ$$

N-VERSION WITH MAJORITY VOTE

IID INPUT

RANDOM SAMPLE	$v_1(x)$	0	0	1	...
	$v_2(x)$	0	1	0	...
	$\vdots$				
	$v_N(x)$	0	0	0	...

SCORE FUNCTION

$$v(x) = \sum_{l=m}^{N} \sum_{\substack{\text{all} \\ \text{permutations}, i}} v_{i(1)}(x) \ldots v_{i(l)}(x) \, [\, 1 - v_{i(l+1)}(x) \,] \ldots [\, 1 - v_{i(N)}(x) \,]$$

$$P_N = E[\int V(x) \, dQ \,]$$

COINCIDENT ERRORS MODEL

Under the conditions that:

(1) components are selected from a random sample

(2) inputs are selected from a common distribution

$$p_N = \int \sum_{\ell=m}^{N} \binom{N}{\ell} \theta(x)^{\ell} [\, 1 - \theta(x)\,]^{N-\ell} dQ$$

$\theta(x)$ = Intensity Function

Q = Usage Distribution

$$= \int h(y;N)\, dG$$

$$h(y;N) = \sum_{\ell=m}^{N} \binom{N}{\ell} y^{\ell} [1-y]^{N-\ell}$$

$$G(y) = \int_{\{x\,:\,\theta(x) \leq y\}} dQ$$ = Intensity Distribution

A DISCRETE INTENSITY DISTRIBUTION

Suppose $\theta(x) = \theta_i$ for $x \in A_i$

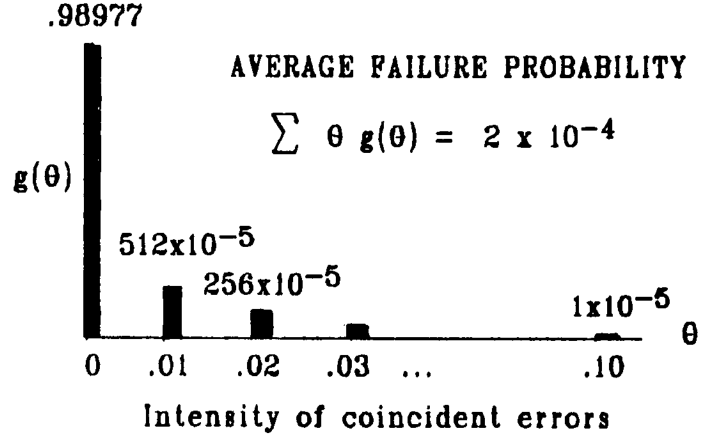
Where $A_1, A_2, \ldots, A_r$ is a partition of Ω

Indexing so that

$$0 = \theta_1 < \theta_2 < \ldots < \theta_r < 1$$

$$G(y) = \sum_{\{i \,:\, \theta_i \leq y\}} Q(A_i) \qquad -\infty \leq y \leq \infty$$

e.g



AVERAGE FAILURE PROBABILITY

$$\sum \theta\, g(\theta) = 2 \times 10^{-4}$$

(e.g. on .001% of inputs, expect 10% of population to produce error)

EFFECTS OF COINCIDENT ERRORS

Under what condition does independence hold?

Under the assumption of a constant intensity the Coincident Errors Model implies the Independent Errors Model.

This constant is the average component failure probability (also the mean of the Intensity Distribution).

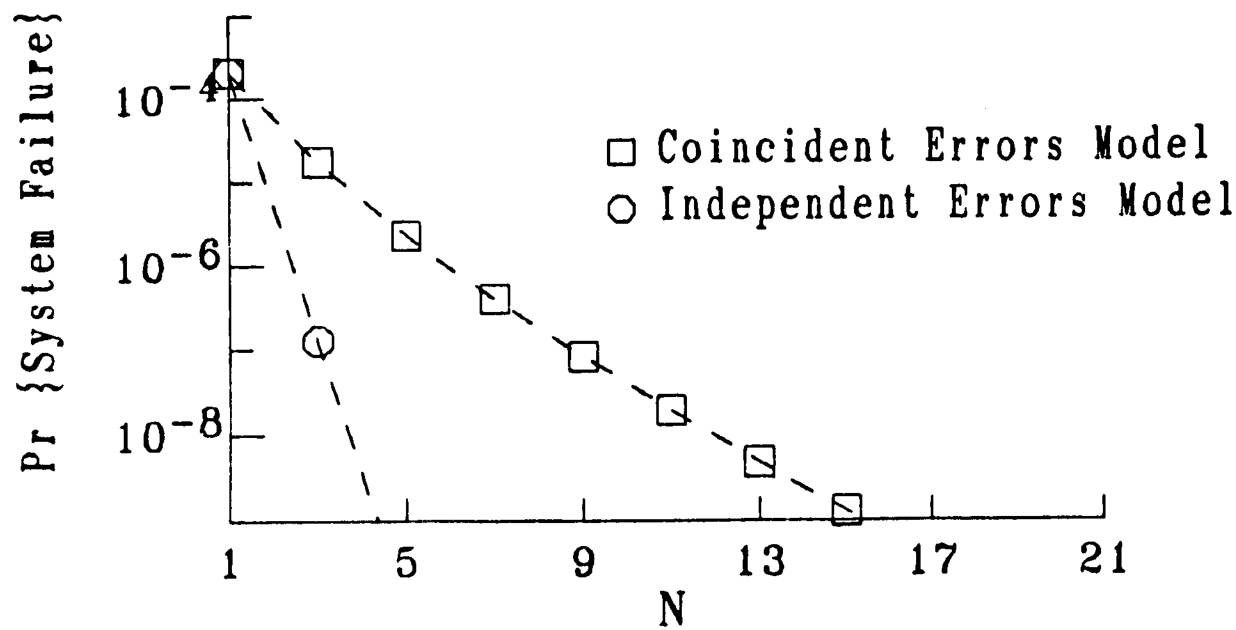
$$P = \int \Theta(x)\, dQ = \int y\, dG(y).$$

EFFECTS OF COINCIDENT ERRORS

Does the Independence Model give a valid estimate of failure probability?

e.g

θ	$g(\theta)$
0	.98977
.01	.00512
.02	.00256
⋮	⋮
.10	.00001

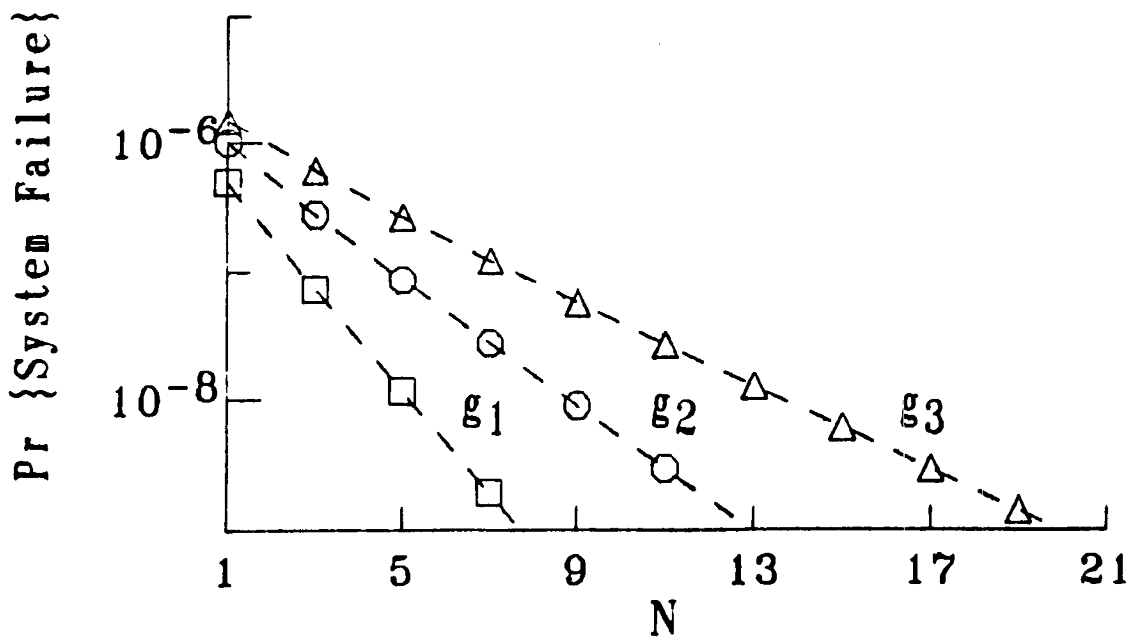


EFFECTS OF COINCIDENT ERRORS

What is the effect of shifting intensity mass probability to the right?

e.g.

θ	$g_1(\theta)$	$g_2(\theta)$	$g_3(\theta)$
0	.99999	.99999	.99999
.05	.00001		
.10		.00001	
.15			.00001

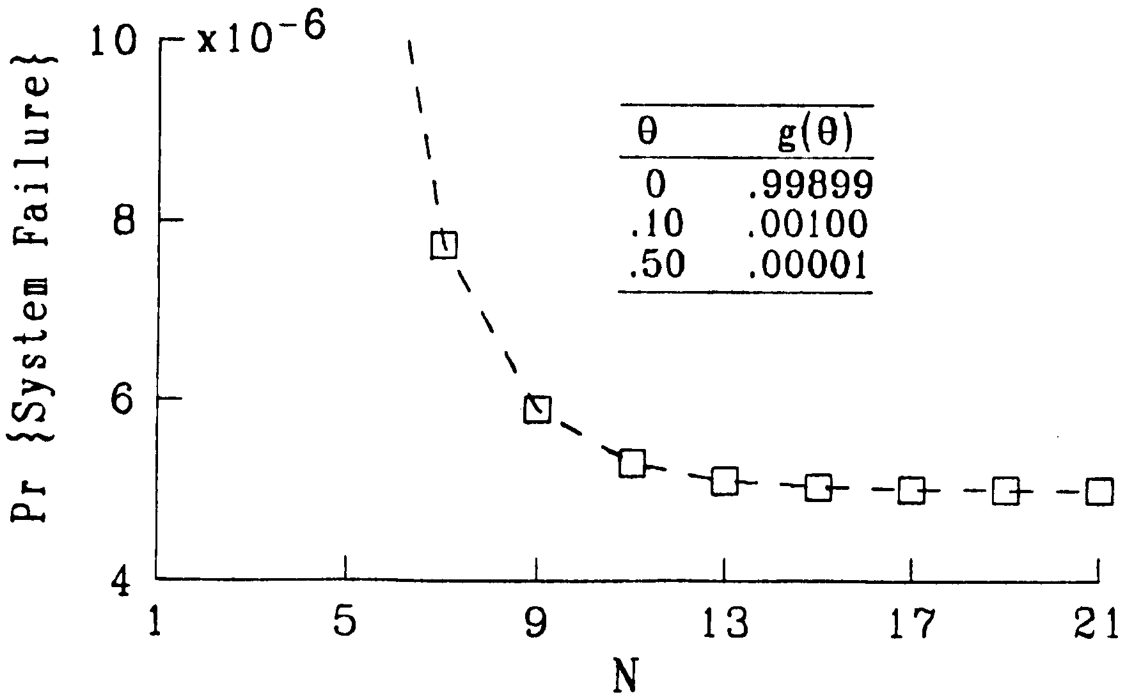


EFFECTS OF COINCIDENT ERRORS

Is there a limit on the effectiveness of redundancy?

Effectiveness limited by mass distributed along inteval $.5 \leq \theta \leq 1$

$$\lim_{N \to \infty} P_N = .5 \, [\, G(.5^+) - G(.5^-) \,] + \int_{.5^+}^{1} dG(\theta)$$



θ	g(θ)
0	.99899
.10	.00100
.50	.00001

EFFECTS OF COINCIDENT ERRORS

Under what condition is an N–Version strategy better than a single version choosen at random?

We say an N–Version strategy is better if $P_N < P$

Where

$$P_N = \int \sum_{l=m}^{N} \binom{N}{l} \theta(x)^{l} \; (1 - \theta(x))^{N-l} \, dQ$$

$$= \int h(y;N) \, dG(y)$$

and

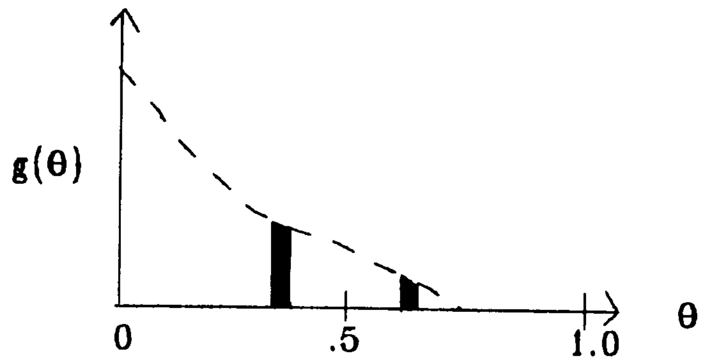
$$P = \int \theta(x) \, dQ \quad = \int y \, dG(y)$$

EFFECTS OF COINCIDENT ERRORS

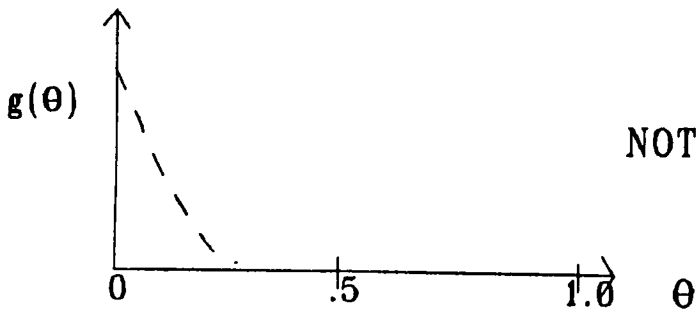
If the asymetry condition,

$$\int_{(y,y+\Delta]} dG \geq \int_{[1-y-\Delta,1-y)} dG,$$

holds for y, $y+\Delta < .5$, then $P_N < P$.

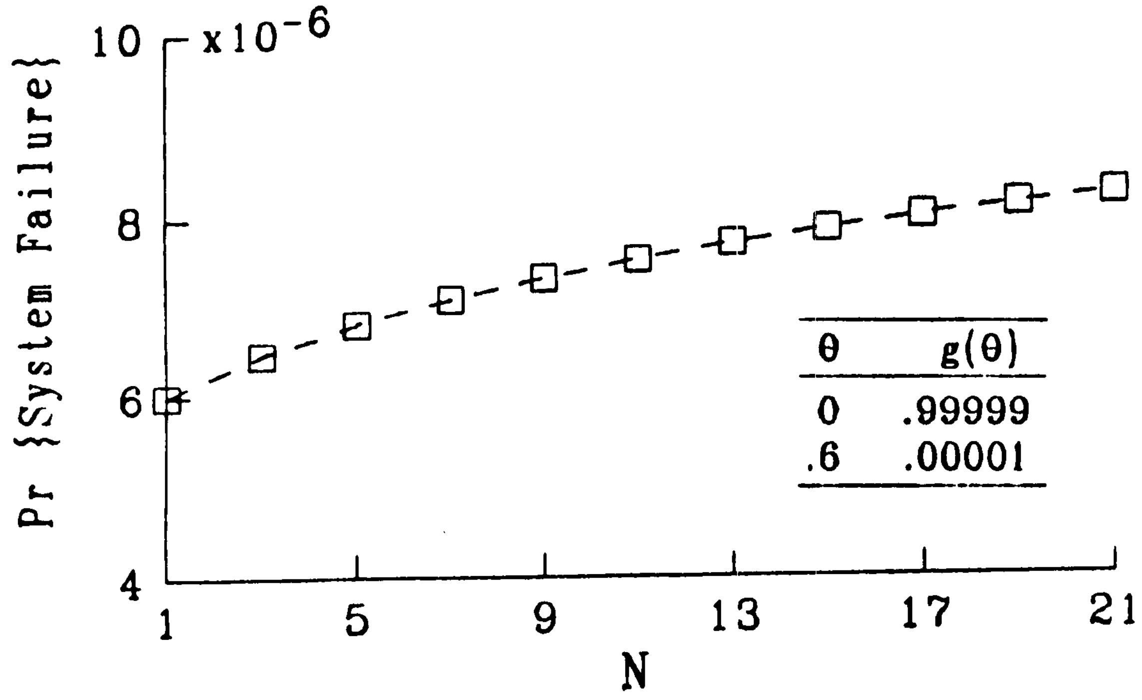


In particular, the asymmetry condition holds whenever the Intensity Distribution is limited above by .5.

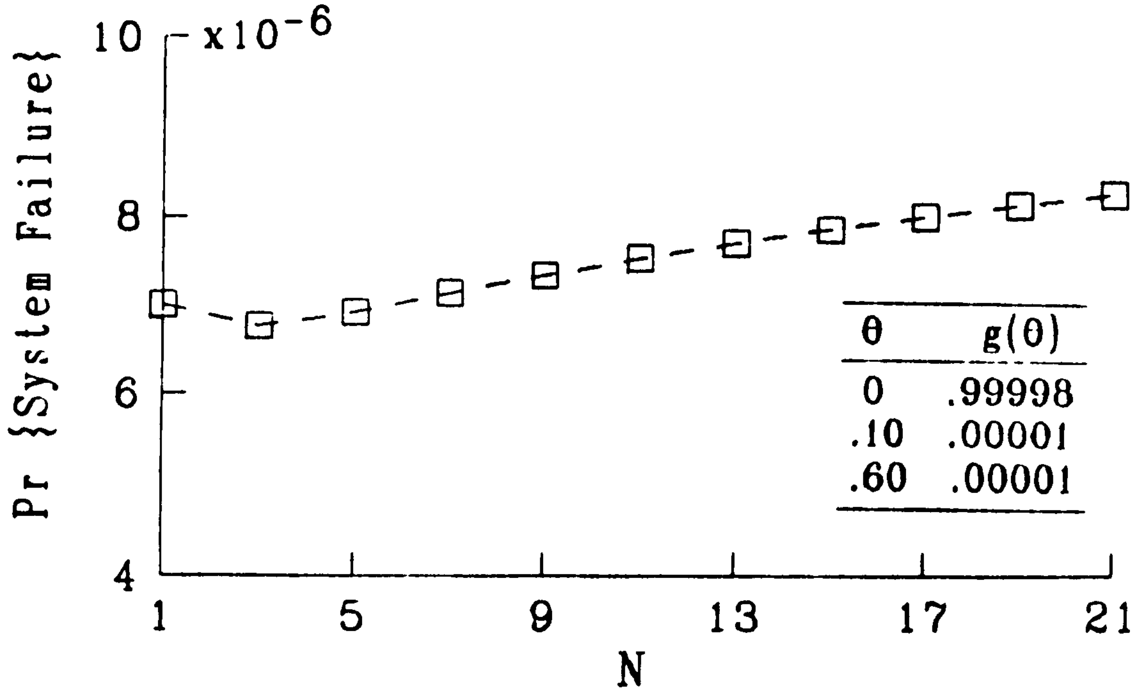


NOTE: $P < .5 \not\Longrightarrow P_N < P$

EFFECTS OF COINCIDENT ERRORS



EFFECTS OF COINCIDENT ERRORS



θ	g(θ)
0	.99998
.10	.00001
.60	.00001

EFFECTS OF COINCIDENT ERRORS

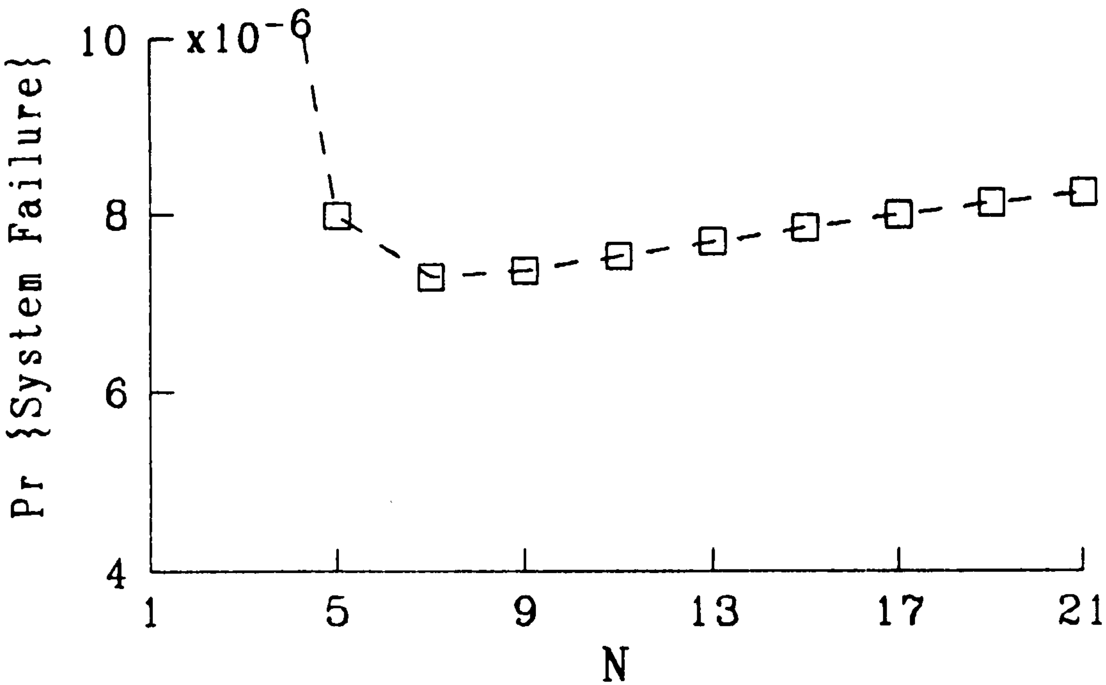
Might an optimum value of N exist?

A necessary condition for system degradation in the limit is a violation of asymetry condition.

e.g.

θ	$g(\theta)$
0	.99899
.05	.00100
.60	.00001

Note: asymetry condition not necessary for $P_N < P$



RESULTS

- Provides a probabilistic framework for assessing strategy of redundant software.
- Provides foundation for experimental study of coincident errors.
- Permits an analytical study to increase understanding of impact of coincident errors.